POSITIVE PROFITS AND POSITIVE SURPLUS LABOUR

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Abstract

In 1975 Ian Steedman gave an example of a pure joint production system (i.e., without fixed capital or land) in which the rate of profits and their associated prices of production were positive and yet aggregate "surplus value" was negative. This example sparked a controversy related to the questions of how to define labour values in the context of joint production and, particularly, whether or not it implied a refutation of the so-called "fundamental Marxian theorem". Many authors tried to get around Steedman´s critique through redefinitions of the labour values of single commodities, arguments about heterogeneous labour and considerations about the problem of choice of technique. Few contributors, however, discussed the actual economic meaning and relevance of the special assumptions in Steedman´s example. In this paper we try to do that by a critical survey and a theoretical evaluation of this literature. We make use of Steedman´s original numerical example as basis to compare and contrast the different contributions. Our main conclusions are that although labour values for some single commodities can indeed be negative in the most general case (this may happen if the system is not all-productive), the basic and sensible classical proposition that in the aggregate less labour would be necessary to produce only the necessary wage basket than to meet also the total final demand coming from the expenditures of the capitalists always holds, even if in some cases producing the necessary wage basket could entail also jointly producing some extra outputs unneeded by the workers.

I. INTRODUCTION

In an article published in the Economic Journal in 1975 Ian Steedman gave an example of a pure joint production system (i.e., without fixed capital or land) in which the rate of profits and their associated prices of production were positive and yet aggregate "surplus value" was negative. This example sparked a controversy related to the questions of how to define labour values in the context of joint production and particularly whether or not it implied a refutation of the so-called "fundamental Marxian theorem". This "theorem" states that a positive "rate of surplus value" – the ratio between the quantity of embodied labour on the physical surplus ("surplus value") and the quantity of embodied labour in the necessary consumption of workers ("variable capital") - is a necessary and sufficient condition for a positive general rate of profits. Many authors tried to get around Steedman´s critique through redefinitions of the labour values of single commodities, arguments about heterogeneous labour and considerations about the problem of choice of technique. Few contributors, however, discussed the actual economic meaning and relevance of the special assumptions in Steedman´s example.

In this paper we try to do that by a critical survey and a theoretical evaluation of this literature. We make use of Steedman´s original numerical example as basis to compare and contrast the different contributions. Our main conclusions are that

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although labour values for some single commodities can indeed be negative in the most general case (it *may* happen if the system is not all-productive), the basic and sensible classical proposition that in the aggregate less labour would be necessary to produce only the necessary wage basket than to meet also the total final demand coming from the expenditures of the capitalists always holds, even if in some cases (as in the particular economic system depicted by Steedman) producing the necessary wage basket could entail also jointly producing some extra outputs unneeded by the workers.

Using a method proposed by Akyuz (1983), it will be shown that it is always possible to find an economic meaningful (i.e., that only deals with non-negative levels of activity) amount of aggregate surplus labour for the economy without changing the processes in use (i.e., using the dominant techniques in use in the square system). As it will be seen, this method seems to be the only available one which is coherent with the idea of taking as given the processes of production in use to measure the aggregate amount of surplus labour.

The paper is organized as follows. Section II briefly discusses issues related to labour values in the context of joint production since Sraffa. Section III presents and discusses Steedman´s assumptions and results. Section IV reviews the debate on positive profits and negative surplus value. Section V offers concluding remarks.

II. LABOUR VALUES AND JOINT PRODUCTION

Labour values, in the case of homogeneous labor, are the physical quantities of labour directly and indirectly necessary to produce a unit of a particular commodity using the dominant (or socially necessary) methods of production actually in use. As it is well known, Sraffa (1960) has shown, for single production systems, that the set of prices of production measure in labour commanded (i.e., divided by the money wage) coincide with the set of labour values when the rate of profits is zero.

In the general case of joint production Sraffa points out that there is naturally an obvious difficulty in thinking of what is the quantity of labour directly and indirectly necessary to produce a particular single commodity, since more than one commodity can be produced by the same process and at the same time that same commodity may be produced by other processes of production.

Some authors such as Schefold (1989) have claimed that the old classical economists and Marx implicitly only thought of single production. But this is clearly a bit exaggerated and difficult to square with the evidence that classical economists dealt with at least some special cases of joint product systems such as differential rent, the treatment of fixed capital as a joint products and even some notions of free disposal and the analysis of industrial byproducts.[1]

It is probably more reasonable to think that, while classical economists did discuss some aspects of joint production, they overwhelmingly seem to have assumed that, even if some elements of joint products were present, it was usually possible to produce single commodities separately, i.e., to increase the net output of a particular commodity without necessarily increasing the net output of any other.[2]

[1] On fixed capital as a joint product in the classical economists and Marx see Gehrke (2012). On pure joint production in the work of the old classical economists and Marx see Kurz (1986).

[2] These systems are variedly known as “all-productive” (Schefold, 1989), or having “weak joint production” (Abraham-Fois & Berrebi, 1997), or having the “adjustment” property, and by adding the assumption of constant returns to scale also, as having the “non-substitution” property (Bidard & Erreygers, 1998).

As we now know, in general joint product systems in which this is possible, it also possible to calculate precisely what is the quantity of labour directly necessary to produce only a unit of that particular commodity and thus there is no difficulty in calculating its positive and additive labour value.

The difficulties with the concept of labour value of individual commodities thus seem to be related only to systems in which production is not separable. And in general the necessary (but not always sufficient) condition for non separability is that there are processes in use that produce net outputs of more than one commodity. That seems to be why simple systems with non-shiftable fixed capital or the analysis of land and differential rents tend to behave like single product systems. Things thus tend to get more complicated within pure joint product systems that do not possess the property of separability.

In these type of systems Sraffa (1960, paragraph 70) has shown that the labour values of individual commodities could turn out to be negative. Sraffa explains the meaning of such negative labour values by referring to the fact that labour values are one and the same thing as the employment multipliers of a single commodity associated to that system of production. A negative labour value thus means that if we think of increasing the net product of only that particular commodity by one unity, and production is not separable and thus we will also increase the net output of at least one other commodity, we will necessarily have to increase the amount of labour employed by one process but also will have to decrease the level of employment in at least some other process to prevent the overproduction of the second commodity. Now depending on the direct amount of labour employed in the process that is expanded compared with the process that is contracted, it may happen that in the end the total amount of labour employed in all processes will be lower than it was before the increased of the net output of the first commodity. In this case the commodity in question will have a negative labour value because an increase in its production has led to a decrease in total (direct and indirect) employment.

Moreover, Sraffa also warned in a footnote[3] of the possibility of the awkward occurrence of what he called “negative industries” by which he meant that the fact that since under joint production the activity levels of some processes must be decreased when that of the others increase to match the level of composition of the “requirements of use” it was logically possible, if the change in the level and structure of demand was sufficiently drastic, that the contraction of some particular joint processes of production could be so extreme as to require it to be activated at a negative level.

III. STEEDMAN´S NUMERICAL EXAMPLE

Steedman (1975) seems to have taken this route and, by inverting the assumptions that Sraffa used to rule out processes with negative activity levels, produced his own particular example. The controversy after the publication of the “positive profits with negative surplus value” example seems to have been due to the combination of two main factors. The first is that Steedman did not give much explanation of the economic meaning of his results nor of the relevance of its assumptions. On the other hand, with very few exceptions, his critics seemed so concerned to “defend” the labour theory of value in some form that they tended mostly to add further ad hoc assumptions to those

[3] Sraffa (1960, footnote 1, paragraph 66).

of Steedman´s example in order to change its conclusions, instead of trying to understand the nature of the example and its implications.

Steedman (1975, p.115) assumes an economy capable of producing a surplus of two commodities using two joint production processes that use only circulating capital:

INPUTS						OUTPUTS		
commodity 1		commodity 2		labour		commodity 1		commodity 2
5	⊕	0	⊕	1	→	6	⊕	1
0	⊕	10	⊕	1	→	3	⊕	12

He also assumes that the wage is paid at the end of the production period and that the real wage basket is exogenously given: **b=** (0.5, 0.83). This assumption is not necessary for the present purposes – it does not matter for the results if it is paid ex-ante or ex-post.

The really crucial assumptions implicit in his analysis amount to three:

(i) A two commodities square system in which both processes generate the same joint products.

(ii) There is a process which is strictly superior to the other one, the second one, having higher net products for both goods[4]. In Steedman´s original example we have the following net products of each process (operated at unity levels): $\mathbf{m_1}$=(1, 1) and $\mathbf{m_2}$=(3, 2) , the columns of the matrix **(B-A).**

These two assumptions imply the *individual* negative labour value for the good one. The third assumption is:

(iii) The proportions in which commodities one and two are demanded by the capitalists are very different from the proportions in which they appear in the wage basket.

This will mean that no combination of the two two processes available is capable of producing without overproduction either only the wage basket or only the profit earner´s basket. As we shall see it is this latter assumption of the unfeasibility of producing the baskets of the two classes separately plus the great divergence between the bundles, not the mere existence of a negative labour value for one of the commodities, which is the key for the occurrence of negative *aggregates* of labour.

Steedman (1975, p.115) first shows that in this particular joint production system then which the rate of profits and relative prices of production are positive, once the profit rate just depends on the existence of a positive net product (surplus product) after the deduction of the necessary consumption for being positive – independently of the labour value measurement.

[4] Wolfstetter (1976) has pointed out that in a two-commodity model, negative individual labour-values can only happen if, and only if, there is a strictly inferior process. However, Hosoda (1993) shows that in models with more than two-commodities, individual negative labour values can occur even without the presence of strictly inferior processes, as defined by Wolfstetter (1976).

From that, he shows that one of the two commodities has a negative labour value:

$$(1)\quad \begin{aligned} 5\lambda_1 + 1 &= 6\lambda_1 + \lambda_2 \\ 10\lambda_2 + 1 &= 3\lambda_1 + 12\lambda_2 \end{aligned}$$

Where λ_1 and λ_2 are the labour values of each commodity. The solution of this system gives us the following vector of labour values:

$$(2)\quad \mathbf{\Lambda = a_L (B - A)^{-1}} = (-1, 2)$$

Where $\mathbf{a_L}$ is the direct labour requirements (composed only by units, given the normalization presented in the table above) and Λ is the additive vector of labour values (which also corresponds to the relative prices associated to the profit rate equal to zero using the wage rate to normalize them)

Then he moves on to show also that the aggregate amount of surplus value in this system is negative, both measured as the aggregate labour value of the sum of commodities demanded by the capitalists and as the difference of total labour employed versus the aggregate labour employed (embodied) in the wage basket.

The total net product is $\mathbf{y}$ is the sum of each class bundle $\mathbf{y_k}$ = (5, 2) which is the final demand which is appropriated by the capitalists, and, $\mathbf{y_w}$ = (3, 5), the workers´ consumption.

To produce this final demand

$$(3)\quad \mathbf{y} = \mathbf{(B\text{-}A)x} = (8,7),$$

a feasible combination of both processes is required:

$$(4)\quad \begin{aligned} x_1 + 3x_2 &= 8 \\ x_1 + 2x_2 &= 7 \end{aligned}$$

Where x_1 and x_2 are the levels of activity of each process required to produce the net product (the components of vector of activity levels). The solution of this system gives the following vector:

$$(5)\quad \mathbf{x = (B - A)^{-1} y} = (5,1)$$

The aggregate amount of labour is given by the sum of each element of activity level vector using this normalization:

$$(6)\quad L = \mathbf{a_L x} = x_1 + x_2 = 6$$

If we multiply the final demand baskets of each class by the labour values of each commodity, this particular technology and pattern of final demand will give us the following labour value aggregates:

$$(7)\quad \begin{aligned} V &= \mathbf{\Lambda y_w} = \mathbf{a_L}\left(\mathbf{B-A}\right)^{\mathbf{-1}}\mathbf{y_w} = (-1).3+(2).5 = 7 \\ S &= \mathbf{\Lambda y_k} = \mathbf{a_L}\left(\mathbf{B-A}\right)^{\mathbf{-1}}\mathbf{y_k} = (-1).5+(2).2 = -1 \end{aligned}$$

Where S is the surplus value and V is the variable capital. The other way for calculating the surplus value is looking at the difference between the living labour (6 units) and the variable capital (7 units):

$$(8)\quad S = L - V = 6 - 7 = -1$$

Before moving on, we think that we can understand this result better if we make the calculation in terms of activity levels associated to the production of each bundle, instead of just finding the labour value of each bundle, as we have done for the whole net product above. For the workers´ bundle

$$(9)\quad \mathbf{y_w} = \mathbf{(B\text{-}A)x_w} = (3,5)$$

we need to solve:

$$(10)\quad \begin{aligned} x_{w1} + 3x_{w2} &= 3 \\ x_{w1} + 2x_{w2} &= 5 \end{aligned}$$

Which gives us the following "activity levels" required to produce workers´ consumption:

$$(11)\quad \mathbf{x_w} = \mathbf{(B-A)^{-1}y_w} = (9,-2)$$

The sum of each component of $\mathbf{x_w}$ gives us the variable capital[5]. Applying the same procedure for the capitalists´ bundle we get:

$$(12)\quad \mathbf{x_k} = \mathbf{(B-A)^{-1}y_k} = (-4,3)$$

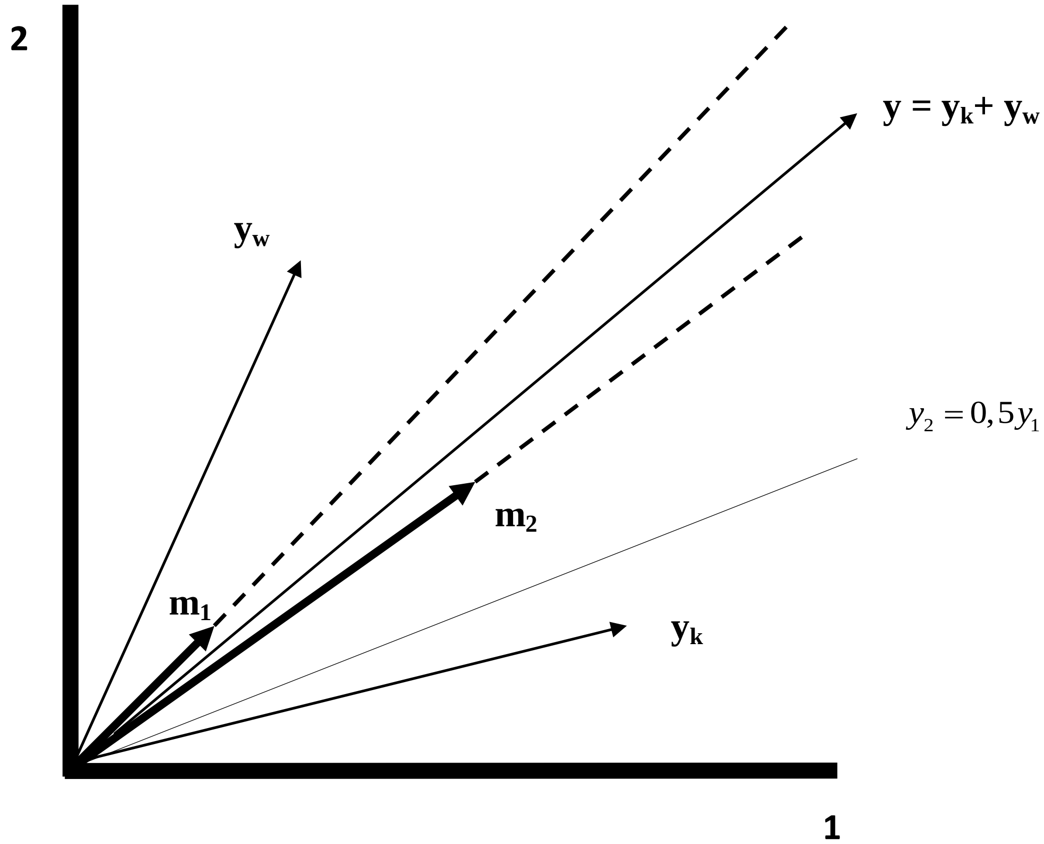
Thus, the surplus value S can be calculated from the sum of the components $\mathbf{x_k}$.

This way of calculating shows that both bundles require negative activity levels of one of the processes because both bundles cannot be produced *separately* by the processes in use[6].

Grafically, we have that:

[5] Steedman (1977, p.159) and Kurz (1979, p.68) also point out the fact that negative levels of activity are "required" to produce separately the workers´ bundle.

[6] It means that if the final demand were one of these bundles alone, the choice of technique would have to change because it would be inefficient to match these final demands using the same square system.



The dotted lines provide the set ("cone") of feasible net products using the two processes given by the vectors $\mathbf{m_1}$ e $\mathbf{m_2}$. The horizontal axis represents the net product of first commodity and the vertical axis the the net product of the second commodity.

As we can see, the total final demand **y** falls inside the cone, which means that it can be reached with non-negative levels of activity. The total employment required for that is 6 units. However, the bundles that each class receives fall outside the cone. This means that they cannot be produced *separately* – though it ´s feasible to produce both jointly. The consequence of this is that negative levels of activity are required to produce only workers consumption or only the final demand of capitalists. If both bundles were inside the cone no negative surplus value could happen, although the labour value of one of the two commodities would still be negative. Indeed, with the same aggregate final demand **y**=(8, 7) but with the consumption of the workers being y_w=(3, 2.5) and the expenditures of the capitalists being $\mathbf{y_k}$=(5, 4.5), for example, we would still get, overall activity levels **x**=(5, 1), total employment *L*=6. But now $\mathbf{x_k}$=(3.5, 0.5) and thus *S*= 3.5+0.5=4 and surplus value would be positive. For the workers we would have $\mathbf{x_w}$ = (1.5, 0.5) and *V*=2 and thus *S*= 6- 2=4.

In the example for the capitalists´ bundle we have that the total employment required to produce it is negative, i.e., negative surplus value. So, we may say that within the group of final demands that are non-feasible, there is a group which "requires" negative *amount* of employment for being produced, which in this numerical example[7] are those ones that lie below the line y_2=0,5y_1. But the central point we want to emphasize is that any final demand outside the cone is economically meaningless[8].

We can thus see that the existence of negative labour values for single commodities doesn´t by itself imply the paradox. The paradox of positive profits and negative surplus value provided by Steedman (1975) rests on the fact the non-feasible levels of activity would be required if the bundles were to be produced separately – or, that these bundles can only be produced jointly within this square system.

Of course negative activity levels do not exist. They are just the mathematical symptom of the fact that there would be overproduction of one of the two commodities if we were to produce only the wage bundle of the workers using these two processes. So the right conclusion from Steedman´s example is that in general joint production systems the calculation of aggregate surplus labour should be adapted to deal with the actual possibility of overproduction, instead of the economic meaningless notion of negative activity levels

IV. SURPLUS VALUE AFTER STEEDMAN

An initial reaction to the Steedman´s example came from Morishima (1976). Based on his earlier works such as Morishima (1973) and Morishima (1974), the author proposed to redefine the labour values in joint production using what he calls the "true values". The "true value" of a commodity (or a bundle) is given by the minimization of labour-time required to produce one net unity of it. Be $\mathbf{e_i}$ the column vector in which the i-th coordinate is unity and the other ones are null, the true-value of the commodity i will be given by the following minimization problem:

$$\text{(13)} \quad \min. \ \mathbf{a_L x}$$
$$\text{s.t. } \mathbf{Bx} \geq \mathbf{Ax} + \mathbf{e_i}, \ \mathbf{x} \geq \mathbf{0}$$

Where activity levels and the processes in use are the endogenous variables of the problem[9] and the symbol $\geq$ means that the vector is equal or higher than the other one. To find the "true variable capital" $\mathbf{e_i}$ must be substituted by $\mathbf{y_w}$, which is going to be the total employment required to produce workers´ consumption which minimizes the amount of labour expended.

Be $\mathbf{x_w}$***** the activity levels that solves the problem, the "true variable capital" would be:

$$\text{(14)} \quad V^* = \mathbf{a_L x_w}^*$$

And the "true surplus value":

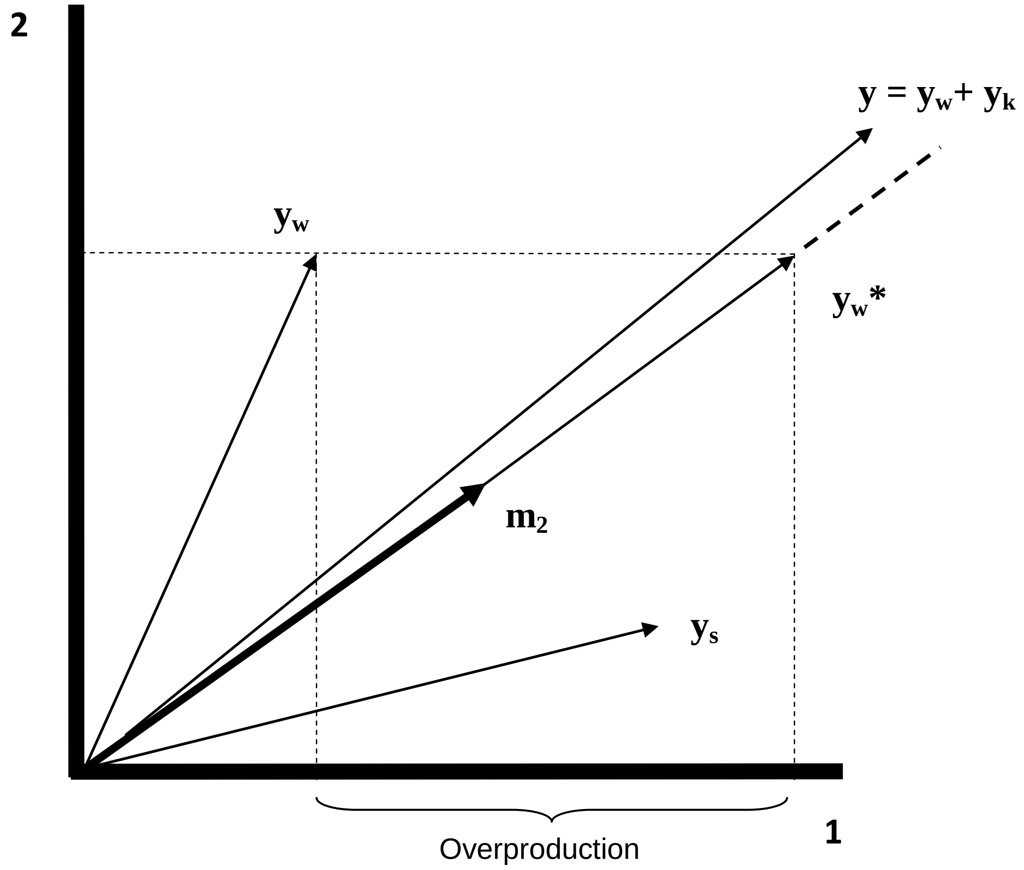
$$\text{(15)} \quad S^* = L - V^*$$

[7] The sufficient conditions for a bundle to have negative aggregate amount of employment in a general two commodity system (that is, being below the line in the graph) and its economic meaning are derived in the appendix.

[8] For example, if the final demands were $\mathbf{y_k}$=(5, 3) and $\mathbf{y_w}$=(3, 4), we would have positive values for *S* and *V*, the same total employment of 6 units and it would seem that no paradox happens. However, both bundles fall outside the cone – activity levels would be $\mathbf{x_k}$=(-1, 2) and $\mathbf{x_w}$=(6, -1) - which means that, despite being positive, either variable capital or surplus value have no economic meaning.

[9] Matrixes **A** and **B** would not be square in the general case**.**

Using Steedman´s example, graphically it would be:



Where $\mathbf{y_w}$***** is the net product associated with the level of activity $\mathbf{x_w}$***.**

In this case, only the process with higher productivity would be operated (this is why $\mathbf{y_w}$***** lies above the same line as $\mathbf{m_2}$). Because operating this process alone cannot produce exactly the workers´ consumption, there would be excess production of the first commodity of 4,5 units. But the point is that less labour than the total employment would be required to produce workers´ consumption. To produce $\mathbf{y_w}$=(3, 5) it would be necessary to operate only the second processes with 2,5 units of labour. This gives a positive "true surplus value" of 6-2,5=3,5.

The criticisms of this procedure are not new: "true-values" are not additive like in Marx, i.e., the "true-value" of a bundle of commodities is not equal to the sum of "true-values" of the same commodities separately produced. The authors argue that this different definition of labour value would have a textual basis in Marx but the argument is not very convicing (Steedman, 1976a). A second important criticism is that in this case the (redefined) surplus value would be related to a non-capitalist (i.e., nonprofit-maximizing) criterion for the choice of technique – while in Marx and in the literature related to the "fundamental marxian theorem" this was based on the processes in use in capitalism ("the socially necessary techniques") (Akyuz, 1982). In fact the "true value" calculations measure the hypothetical minimum amount of labour that would be necessary to produce the wage basket for society as such and not the amount that is "socially necessary" given the techniques and processes actually already chosen by a capitalist criterion of choice of technique.

Wolfsetter (1976) basically questioned Steedman´s use of Sraffa´s approach of assuming square joint production systems with equalities, instead of starting from Von Neumann´s method of starting from rectangular systems and using inequalities. Steedman (1976b) promptly replied that in the particular case of his example the difference between the two methods would hardly matter for his results, the solution being exactly the same for both approaches. Besides, the author´s third theorem (Wolfstetter, 1976, p.867), which states that the existence of an inferior process is a necessary and sufficient condition for negative individual labour values, is correct only in a two commodity system as pointed out in the footnote 3.

Kurz (1979) and Krause (1980) tried to refute the paradox redefining the vector of individual labour values to make it semi-positive. Krause (1980) argued that the different labour productivities are a case of heterogeneous labour and with this argument the author changes the weights of the direct labour vector in such way to guarantee strictly positive individual labour values. The obvious problem with this complicated solution is that the assumption of heterogeneous labour is simply not present in Steedman (1975).

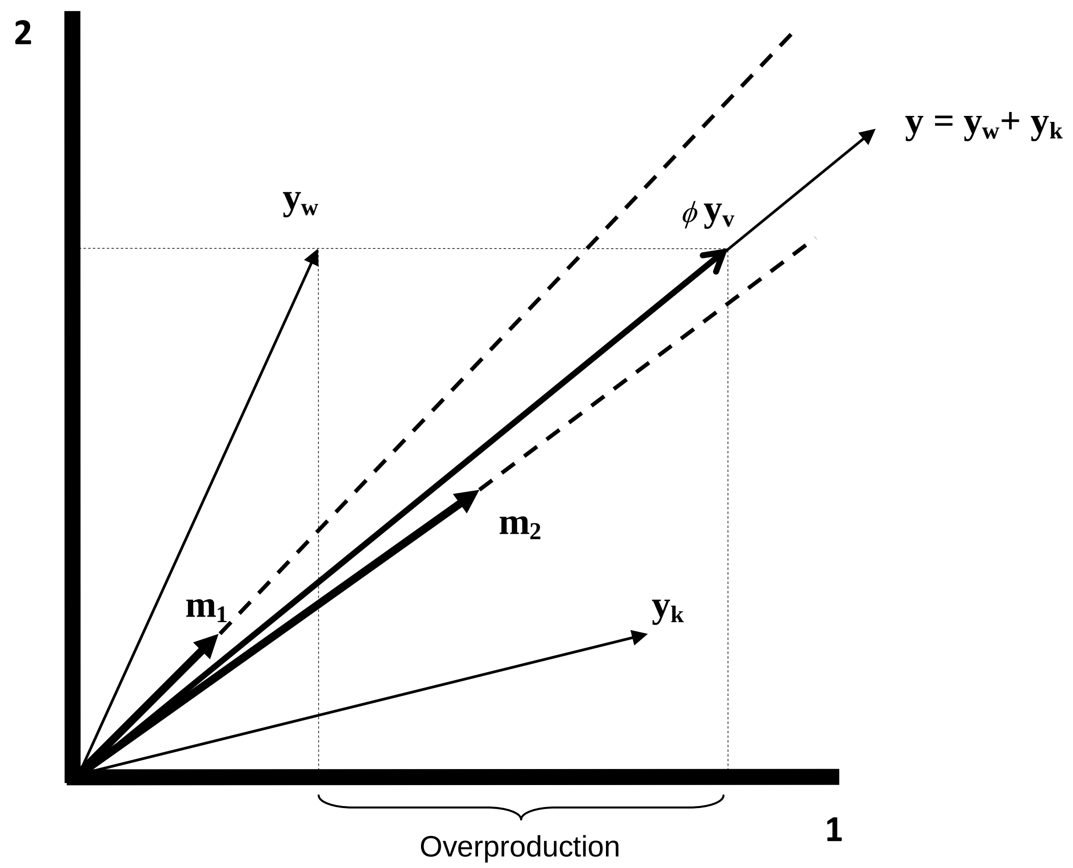
Akyuz (1982) has provided what seems to be the least ad hoc way to find an economic meaningful measure for necessary employment. The author proposes a method in which a bundle of goods at least equal to $\mathbf{y_v}$ is produced using the processes actually in use and keeping the same proportion in which they are actually operated. This scale of production is a scalar ϕ given by the following minimization problem[10]:

$$(16)\ \text{Min } \phi$$
$$\text{s.t. } \phi\mathbf{y} \geq \mathbf{y_w}; \phi \in (0,1)$$

Where $\mathbf{y}$ is the actual net product of the system and $\mathbf{y_w}$ is the actual workers´ consumption basket. The symbol $\geq$ means equal or higher. Grafically, we would

[10] Akyuz actually says that there´s no need for a minimization problem and that it would be the great virtue of the method (Akyuz, 1982, p.171). However, it seems to the present authors that this is not complete because Akyuz (1982, p.178) puts the problem in the following way: there is a scalar ϕ such that $\phi \in (0,1)$ and that $\phi y \geq y_w$. However, in this case it may happen that there are many scalars that satisfy the inequality and we would have an indeterminacy. It seems to us that the insight would be complete if presented as a minimization problem of finding the minimum scalar. What he probably meant is that there is no need for changing the processes that are already in use.

have:



We can see that the vector falls inside the cone and, consequently, a positive amount of employment is required to produce it. In this case, we would have a redefined vector of workers´ consumption which has the advantage of being based in the processes actually in use and keeping the same proportion in which they are actually operated. This redefined surplus value *S'* will be given by:

$$S' = (1-\phi)L \quad (17)$$

If we make a comparison with the "true-values" we can see that this method requires much less new information and is much closer to the actual system data than the former. In the procedure proposed by Morishima (1976) the processes and their relative levels of activity are altered (besides the presence of non-profit maximizing criterion for the choice of technique). In the method proposed by Akyuz (1982) there´s only the scalar as novelty and no change in the choice of techniques is required. Inevitably (as in the "true-value" approach) there´s some excess of production – of the first commodity (2,174 units in the example).

In any case Akyuz´s method allows us to see that the classical proposition that (without changing the actual processes that are in use in a capitalists) workers obviously work more than they would need to produce only (at least) what is contained in their wage basket. This proportion relating positive profits and positive surplus labour is of general validity with or without joint production, and whatever may be happening to the labour value of individual commodities.

Flaschel (1983, p.437) adds relative prices in the definition of labour values in order to get positive magnitudes for the latter. This seems to be a contradiction with the original function of the labour values of being a measure of physical costs, independent from (and a major determinant of) income distribution and relative prices

More recently Trigg and Philp (2008) have claimed that there will always be positive surplus value in aggregate even in the presence of individual negative labour values for single commodities. Based in our argument, it seems that Trigg & Philp (2008) implicitly assume that the wage basket falls inside the "cone" and thus see no problem in calculating positive surplus labour – otherwise the Steedman´s paradox could happen. In other words, they seem to assume that their system can produce separately the wage basket through a combination of strictly positive activity levels of the processes of production in use but such assumption should be made explicit to show the logical limits of the validity of their analysis.

V. CONCLUDING REMARKS

The present work has proposed to give a clarification of the economic properties behind the "positive profits with negative surplus value" example provided by Steedman (1975). Criticisms such as the lack of economic meaning of the presence of an inferior process seem quite misleading because the negative marxian aggregates are not a direct consequence of it[11].

Individual negative labour values are certainly a necessary condition for the paradox to happen but the central point seems to be the fact that the bundles that go to each class must: (i) fall outside the cone given by the square system[12] and (ii) have a very different composition. So, the negative surplus value of the example is related to the fact that non-feasible levels of activity would be required to produce only (with no overproduction) the capitalists´ bundle – something that is devoid of economic meaning.

As the alternative provided by Morishima´s true values doesn´t seem useful as being too "normative" we think that the method provided by Akyuz (1983) seems to be the simplest and least arbitrary way to get an economic meaningful measure of surplus labour in this case. The procedure has the advantage of using only data from the methods that are actually in use as opposed to the literature on the so called "fundamental Marxian theorem".

Akyuz´s method surely seems a possible way to relate positive profits with aggregate surplus labour but it is still true that it is still a bit cumbersome in cases like the one used in Steedman´s example because of the unavoidable issue of overproduction[13]. Perhaps these complications were behind Sraffa´s (1960) procedure of abandoning all labour embodied measures and using the amount of labour commanded

[11] As pointed out in the footnote 2, although the existence of inferior process is a sufficient condition for individual negative labour-values to occur (in 2x2 is a necessary and sufficient condition), it may happen also in other cases (Hosoda, 1993).

[12] Certainly, if there were many techniques – rectangular systems – any final demand could be matched and no paradox would happen, but this would be very far from the original debates, based on the techniques in use.

[13] Some may argue that this method is not "general" in a purely mathematical sense of the word: for example, if we had a final demand of (9, 4) and workers´ consumption of (1, 4) there would be no solution because the scalar that would solve the problem would be unity. As long as the final demand is strictly higher than the workers´ bundle ($\mathbf{y} \gg \mathbf{y}_{\mathbf{w}}$) the method seems to hold.

by his “standard net product” when trying to connect the rate of profits, the physical surplus and aggregate labour in his own book. But that is another story.

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APPENDIX: THE ECONOMICS OF THE UNFEASIBLE BUNDLES IN A TWO-COMMODITY SYSTEM

As mentioned before, to generate negative surplus value (or, more generally, any negative aggregate of labour value) more than unfeasible bundles are required. Although negative levels of activity are a necessary condition it is also necessary a very specific composition of a bundle **y** to obtain the paradox as will be seen in a very simple algebraic demonstration that follows.

As we´ve normalized the system for units of direct labour, we have that the labour value of a bundle **y** is given by:

$$(18)\quad \mathbf{\Lambda y} = \mathbf{a_L (B - A)^{-1} y} = \mathbf{a_L x} = x_1 + x_2$$

Let us define **M** such as:

$$(19)\quad \mathbf{M = (B - A)} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

the inverse of **M** in this case is given by

$$(20)\quad \mathbf{M^{-1}} = \frac{1}{\det \mathbf{M}} \begin{bmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{bmatrix}$$

Provided that det**M** is non-null, we know that:

$$(21)\quad \mathbf{x = M^{-1} y}$$

If we want a negative labour value of the bundle **y** we need to have

$$(22)\quad x_1 + x_2 = (y_1 m_{22} - y_2 m_{12}) + (-y_1 m_{21} + y_2 m_{22}) < 0$$

This gives the following inequality:

$$(23)\quad \frac{y_1}{y_2} > \frac{m_{12} - m_{11}}{m_{22} - m_{21}}$$

Which shows that for having a negative amount of employment necessary to “produce” a bundle we need to assume that *the ratio of the two commodities in a bundle is greater than the ratio of the differences of efficiency in the production of commodities of process one relative to process two*. This means that only the existence of a strictly superior process is not sufficient to obtain a negative aggregate labour value of a bundle of commodities. We need either a very wide disparity in the demand for good one and two and/or a small difference of efficiency between the two processes.